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## Weak Rejection

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### ABSTRACT

Linguistic evidence supports the claim that certain, *weak* rejections are less specific than assertions. On the basis of this evidence, it has been argued that rejected sentences cannot be premisses and conclusions in inferences. We give examples of inferences with weakly rejected sentences as premisses and conclusions. We then propose a logic of weak rejection which accounts for the relevant phenomena and is motivated by principles of coherence in dialogue. We give a semantics for which this logic is sound and complete, show that it axiomatizes the modal logic KD45 and prove that it still derives classical logic on its asserted fragment. Finally, we defend previous logics of strong rejection as being about the linguistically preferred interpretations of weak rejections.

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### 1. Introduction

**Assertion and Rejection.** Frege [1919] famously argued that in analysing a discourse, it is superfluous to differentiate *rejections* (also called *denials*) from *assertions*, as rejection can just as well be understood as negative assertion. Hence, so the story goes, a sign for rejection increases the number of primitives without explaining more phenomena, and should therefore be foregone under maxims of parsimony. The opposing view that rejection and assertion should be treated separately is called *bilateralism*, and Smiley [1996] is the champion of its recent resurgence.

In what follows, assertion and rejection are *speech acts*, which can be understood as expressing *attitudes* towards sentences: assertions express *assent*, whereas rejections express *dissent*.<sup>1</sup> Frege analyses assertion and rejection in dialogue by considering positive and negative answers to questions. Smiley follows suit. He observes that one can realize these speech acts by posing a question to oneself ('Is it the case that A?') and then answering it ('Yes!' to express assent and 'No!' to express dissent). So, an assertion of  $\neg A$  can be realized by (1) 'Is [it the case that]  $\neg A$ ? Yes!' and a rejection of  $A$  by (2) 'Is [it the case that]  $A$ ? No!'.

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<sup>1</sup> Sometimes *rejection* is taken to be the *attitude* expressed by the speech act of *denial*. See, for example, Restall [2005] and Ripley [2011]. This difference is merely in terminology. Frege himself considers two *modes of judging thoughts*: positive and negative.

Smiley maintains that (1) and (2) are genuinely different speech acts, whereas Frege argues that (2) reduces to (1). After reviewing their arguments, we present an explicit example to which Frege's reductive strategy does not apply. This example is linked to another challenge that has been posed to bilateralism: that there are certain, *weak* rejections that are supposedly unsuitable for inference and hence resist bilateralist treatment. We meet this challenge by showing how weak rejections can occur as premisses and conclusions in inferences. We then proceed to develop an account of weak rejection that admits a bilateralist proof theory.<sup>2</sup>

**Frege's Argument.** Frege's argument that rejection is negative assertion goes as follows. He takes the inference in (3) below to be valid natural language reasoning and considers (4) and (5) as possible analyses.<sup>3</sup>

- (3) a. If the accused was not in Berlin, he did not commit the murder.  
 b. Was he in Berlin? No!  


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 c. Did he commit the murder? No!

- |  |  |
|--|--|
| (4) a. Assert: If not $p$ , then not $q$ . | (5) a. Assert: If not $p$ , then not $q$ . |
| b. Assert: not $p$ .                       | b. Reject: $p$ .                           |
| <hr/> c. Assert: not $q$ .                 | <hr/> c. Reject: $q$ .                     |

In (4), the word 'no' in response to 'was he in Berlin?' is taken to *modify* the propositional clause: (4b) expresses *assent* to the proposition that he *was not* in Berlin. In (5), 'no' is interpreted as expressing an attitude the speaker has *towards* the propositional clause: (5b) expresses *dissent* from the proposition that he *was* there.

Now, Frege argues, (4) is validated by *modus ponens*: the antecedent of the conditional in (4a) is asserted in (4b). However, the propositional clause in the antecedent of (5a) is not the same as the one in (5b), so *modus ponens* cannot apply there; Frege also emphasizes that rejections cannot embed under conditionals, so the 'not' in the antecedent of (5a) must indeed modify the propositional clause. Thus, to preserve the validity of (5), the rejection in (5b) must be analysed as the assertion in (4b)—and then we might as well forego the distinction. The following is a possible reconstruction of Frege's argument:

- i. (5) is a candidate analysis of (3).
- ii. (3) is valid.
- iii. The inference from (3ab) to (3c) is an instance of *modus ponens*.
- iv. Hence, *modus ponens* must be applied to (5ab).
- v. Thus, (5b) is (4b).

<sup>2</sup> In the interest of readability, we omit the proof or provide proof sketches for the formal results of this paper. A supplement with the full proofs is available on our websites: <http://sites.google.com/site/lucaincurvati/> and <http://jjsch.github.io>.

<sup>3</sup> Frege seems to use embedded negations and negative answers to questions interchangeably. (3b) is our rendering of Frege's 'the negative [*verneinende*] answer to the question "was the accused in Berlin at the time of the murder?"' [1919: 153].

Now, Smiley points out that (iii) need not be true, so all that Frege shows is that *modus ponens* does not apply to (5) if rejection is distinct from assertion.<sup>4</sup> Smiley [1996: 4] draws attention to the rule in (6).

- (6) a. Assert: If  $p$ , then  $q$ .  
       b. Reject:  $q$ .  
       c. Reject:  $p$ .

This is *prima facie* valid: if I assert a conditional but reject its consequent, I ought to reject the antecedent as well. If we identify rejection with negative assertion, we understand (6) as *modus tollens*, but the bilateralist point is that we do not *need* to do so. Likewise, we do not need to understand (5) as *modus ponens*. Thus, (iii) is grounded in parsimony: maintaining (5b) as distinct from (4b) requires us to add a new primitive for rejection as well as novel inference rules. However, Smiley continues, there is nothing unparsimonious in doing so if this explains more data. Now, Smiley takes (3) to constitute new data, but Frege would insist that such data could also be explained via his reductive strategy. To break this impasse and prove Smiley's point, we present below a case (example (7)) that resists analysis as (4).

**Bilateralism.** Smiley's bilateralism has been given a concise formulation by Incurvati and Smith [2009: 3], who ask us to 'imagine a speech community for whom any sentence is explicitly structured into a propositional content clause and a force-indicator'. In that community, utterances are constructed by *marking* propositional clauses with their *force*.<sup>5</sup> Using + as a marker for assertive force and – as a marker for rejective force, they can analyse (1) as  $+ \neg p$  and (2) as  $-p$ . Some novel inference rules in their logic are:

$$(+ \neg I.) \frac{-A}{+ \neg A} \quad (+ \neg E.) \frac{+ \neg A}{-A}$$

Thus, they validate (5) as follows: from  $-p$  move to  $+ \neg p$ , then apply *modus ponens* to arrive at  $+ \neg q$  and hence  $-q$ .

We want to explore an alternative theoretical option that, to our knowledge, has not received substantial attention yet: account for (5) by an inference that does *not* appeal to the *modus ponens* of (4). To appreciate the merits of doing so, consider the following variant of (3).

- (7) a. If the chair of logic is not here, the chair of metaphysics is not here.  
       b. Is the chair of logic here? No, the position is still open.  
       c. Is the chair of metaphysics here? No.

This still is a valid inference: it is natural in situations in which, for instance, the chairs of logic and metaphysics are in personal union, or the speaker knows that the chair of metaphysics would only come to meet the new chair of logic, once appointed.

<sup>4</sup> In fact, Frege himself concludes that the inference from (5ab) to (5c) cannot be performed 'in the same way as before' [1919: 154], but makes no claim to the invalidity of (5) in general.

<sup>5</sup> In what follows, we will talk about force-marked *sentences*. This is to account for rejections of sentences that might be interpreted as *failing* to express a proposition. See examples (9) and (10) below.

However, the analysis as (4) does not apply here: saying ‘no, the position is still open’ is not equivalent to asserting that *the chair of logic is not here*. Following Frege’s analysis of (3), this means that this is not a *modus ponens* inference, since (7b) does not coincide with the antecedent of (7a). One could analyse (7b) as a negative assertion by giving the negation *external scope* and read (7b) as ‘it is not the case that the chair of logic is here’. However, this does not coincide with the antecedent of (7a) and so does not license *modus ponens* either. To say otherwise, moreover, would obscure the difference between (7b) and the alternative rejection ‘No, she is not’.

Hence, we contend that examples like (7) are better understood bilaterally. That is, given a suitable conception of rejection, it is reasonable to expect that (5) remains a valid analysis of (7), not reducible to *modus ponens*.

**The Weak Rejection Challenge.** Example (7) is linked to the *weak rejection* challenge to bilateralism. It has been argued that bilateralists cannot account for the phenomenon of rejection as a whole because there are cases like (7b) where a rejection appears to be less informative than a negative assertion. Note, in particular, that in such cases the rule  $(+\neg I)$  does not apply. We will call rejections *strong* if  $(+\neg I)$  applies to them and *weak* otherwise.

Dickie [2010] calls the existence of weak rejections alongside strong rejections the ‘messiness’ of rejection.<sup>6</sup> She argues that it puts bilateralists in a bind: either bilateralism is explicitly stated only for strong rejections or it also covers weak rejections. If  $\neg A$  denotes the strong rejection of  $A$ , then the crucial *Smilean reductio* principle  $+A \vdash \perp \Rightarrow \vdash \neg A$  is invalid: if it is absurd to assert  $A$ , then  $A$  may be rejected, but not necessarily *strongly* rejected, or so Dickie argues.<sup>7</sup> If  $\neg A$  can denote a weak rejection, then, says Dickie, the messiness of weak rejection precludes one from giving an evidence-preserving proof theory.

Therefore, Dickie concludes, rejection is too unspecific to be useful in inferences. To assess Dickie’s argument, let us take a closer look at the rejection phenomenon. Consider the following examples:<sup>8</sup>

- (8) Did Homer write the *Republic*?  
No, Homer (a guy who actually existed and wrote the *Odyssey* and the *Iliad*) did not also write the *Republic*.
- (9) Did Homer write the *Iliad*?  
No, in fact, Homer did not exist.
- (10) Was Homer a unicorn?  
No, there is no such property as the property of being a unicorn.
- (11) Is it the case that X or Y will win the election?  
No, X or Y or Z will win.

<sup>6</sup> Such claims have a long history, see Horn [1989: chs 1–2]. The longstanding view can be succinctly put as follows: assertions tell us how the world is; rejections tell us much less.

<sup>7</sup> In fact, Rumfitt [1997] already makes the argument against *Smilean reductio* for strong rejections. He also considers rejections that do not amount to a negative assertion. He calls them *external* rejections (and our strong rejections *internal*); compare with example (7). We will make sense of this terminology in section 4.

<sup>8</sup> Examples (8), (9) and (10) are due to Dickie [2010]; (11) is adapted from Grice [1991]; (12) is our own.

- (12) Will there be a seminar talk tomorrow?  
No, not as far as I know.

Here, the rejecting speaker marks the sentence in question as *unassertible* in one way or another. There are many grounds for finding a sentence unassertible, falsity as in (8) being just one of them—hence ‘messiness’. The same speaker would not necessarily be comfortable with asserting the negated sentence: in many cases, the negated sentence would be unassertible too.

A rejected sentence need not be unassertible *in principle*. In examples (11) and (12), the rejecting speaker *ascribes* unassertibility to the sentence in question, but the sentence does not suffer from faults such as malformedness or presupposition failure. Also, the rejecting speaker does not reject the sentences there as *false*. If the rejecting speaker of (11) were interpreted as saying that it is false that X or Y will win, then they would be interpreted as asserting that Z *is* going to win, but this would be an overstatement. Similarly, the epistemic state the rejecting speaker self-reports in (12) prevents us from interpreting them as asserting that there is *no* such seminar. Thus, the linguistically acceptable rejection of a sentence cannot always be judged by considering the sentence in isolation.<sup>9</sup> Note that despite their alleged messiness, such rejections still plausibly satisfy the inference schemes (5) and (6), as the following instantiations show.

- (13) a. If the election will not be won by X or Y, then we will not have a socialist president.  
b. Is it the case that X or Y will win the election? No, X or Y or Z will win.  
c. Is it the case that we will have a socialist president? No.
- (14) a. If there is a seminar today, Franz is here.  
b. Is Franz here? No, not as far as I know.  
c. Will there be a seminar talk? No.

An appropriate context for (13) is one in which X and Y are the only socialists on the ballot; one for (14) is one in which Franz is chairing the seminar. Thus, as some of our examples show, rejections—including weak ones—may be used as premisses and conclusions in inferences. We aim to give a unified logical framework for inferences involving assertion and rejection. We will first motivate a natural conception of weak rejection that accounts for all the above examples and the inferences they occur in. We will then provide a bilateral system faithful to this conception and investigate its logical properties.

## 2. Weak Rejections

Our goal is to extend the bilateralist approach to weak rejections. We begin by discussing how one can conceive of the effects of rejection in terms of *public commitment*. We then motivate how this conception is a natural complement to accounts of asserted content, and why we are justified in considering the broad variety of rejections as instances of the same speech act.

<sup>9</sup> To our knowledge, this point has been mostly overlooked in the literature on rejection so far. A notable exception is Grice [1991], who draws attention to example (11).

**Commitment and Coherence.** Lascarides and Asher [2009] present the notion of *public commitment* as a driving force behind the dynamics of agreement and disagreement in dialogue. Under that conception, making an assertion has the effect of publicly committing the speaker to the asserted sentence. Hence, by asserting ‘it is the case that *A*’, a speaker undertakes a commitment to *A* in a way that restricts the possible contexts this speaker can be placed in. Lascarides and Asher understand the restriction effected by undertaking a commitment in terms of *dialogue coherence*: the principles by which speakers are permitted to make certain speech acts without sounding odd.<sup>10</sup> To them, coherence is a broad notion, including, for instance, that speakers ought to stay on topic. For our purposes, it suffices to consider only coherence as it relates to commitment. Simply put, if two restrictions on context contradict each other, the dialogue is incoherent. That is, if there are grounds to infer that a speaker is committed to *A* and to infer that they are *not* committed to *A*, then the dialogue is incoherent.

The notion of public commitment gives rise to considerations about logical inference. For instance, we can justify the standard conjunction elimination rule as follows: if I need to be in a context for  $A \wedge B$ , I need to be in a context for *A*—that is, my commitment to  $A \wedge B$  implicitly commits me to *A*. In the next section, we will use this approach to motivate the inference rules of a logic of weak rejection.

**Weak Rejection.** If we conceive of assertion as leading to public commitment, a strong bilateralism takes the rejection of *A* as implicitly committing one to  $\neg A$ . But if rejection is a foil to assertion, there is another option: to take the rejection of *A* as having the effect of publicly announcing that one is *refraining from committing* oneself to *A*.

This option can be motivated on the basis of accounts of assertion centred around its conversational effect. On a Stalnakerian account, the essential effect of an assertion is to propose an update of the context the speakers are in [Stalnaker 1978]. On a commitment account, it is to undertake a public commitment. On both models, successful conversation requires keeping the contexts of the interlocutors aligned. To Stalnaker this means expanding *common ground*; to Lascarides and Asher it means establishing public commitments as *shared*. In both cases, expansion of the shared context requires mutual acceptance. Evidently, not every sentence asserted in a dialogue will be acceptable to all speakers. Therefore, there ought to be a mechanism by which a speaker can stop the update of the context. This is the speech act of weak rejection.

It will not do to reduce this speech act to an operator (like ‘not’ or ‘it is unassertible that’) that allows one to *assert* that one is dissenting from a sentence. In asserting, one proposes to update the context, but we are looking for a way to *not* update the context. That is, we require something that operates dually to assertion, on the same level as assertion. One might attempt to assert the negation of a sentence and thereby prevent that sentence from being added to the context. But then something more than just blocking a context update has happened. We contend that refraining from accepting a sentence is more basic.<sup>11</sup> For, as we will show later, rejection through negative assertion can be linguistically recovered from our notion of weak rejection.

<sup>10</sup> The idea that an assertion results in the undertaking of a commitment goes back to the philosophical literature, notably Brandom [1983]. Lascarides and Asher’s contribution is to relate commitment to dialogue coherence.

<sup>11</sup> Stalnaker appears to be sympathetic to this: ‘If an assertion is rejected, the context remains the same as it was.’ [1978: 87, fn. 9].

The issue remains as to why the broad variety of weak rejections we have seen should all be considered instances of a unique speech act. Note that our motivation for the speech act of weak rejection is grounded in the *mechanisms* surrounding assertion. When these mechanisms are taken into account, the seemingly unrelated instances of rejection turn out to all be cases in which one refrains from committing, with differences only in the *grounds* for doing so. Moreover, all these instances of rejection can be marked with the particle ‘no’. Thus, the most parsimonious interpretation seems to be this: ‘no’ is a force-marker for weak rejection, but the grounds for rejection can vary.

Similar considerations apply to unassertibility: whenever a speaker has grounds to refrain from updating the context with a sentence, we say that that speaker finds the sentence unassertible. Thus, the felicitous rejection of a sentence indicates that a sentence is unassertible to the rejecting speaker. Conversely, if a speaker finds a sentence unassertible, and that sentence is proposed for a context update, the speaker has grounds to refrain from accepting that update and is therefore expected to reject the sentence. Note that one does not need to be engaged in dialogue to pose the question ‘Is it the case that A?’ to oneself. Hence we can say that having the attitude of dissent towards a sentence is tantamount to finding that sentence unassertible, which means to have grounds to answer ‘no’ to that question.

**Related Notions.** Rumfitt [1997] and Restall [2005] develop accounts of rejection which it is useful to compare with ours.<sup>12</sup> Rumfitt contrasts answering ‘No’ with ‘refus[ing] to give the answer Yes’ and claims that the latter expresses dissent ‘in another perfectly good sense’ [Rumfitt 1997: 226]. Restall says that to reject is ‘to *refuse* to accept [assent]’ [Restall 2005: 197, fn. 6, emphasis his]. On our account, assent is expressed through assertion, which is the undertaking of a commitment. So ‘refusing to answer Yes’ and ‘refusing to assent’ (if expressed in a speech act) would have the effect of refusing to commit. This appears to be slightly stronger than our account of weak rejection. The examples (11) and (12) are weak rejections in our sense, but do not seem to fall under Rumfitt’s and Restall’s accounts.

Rumfitt [1997] argues that weak rejections call for a three-valued logic (true/false/neither). In a later paper [Rumfitt 2000], however, he takes a rejection to mark a sentence as ‘other than true’ and says that this notion is broader than his earlier one [Rumfitt 2000: 799–800]. On our account, a rejection need not mark a sentence as having or not having a particular truth-value. The advantage of this approach can be appreciated by considering again example (11), due to Grice.

- (15) If you say “X or Y will be elected,” I may reply “That’s not so; X or Y or Z will be elected.” Here, too, I am rejecting “X or Y will be elected” not as false but as *unassertable*.

[Grice 1991: 82, emphasis his]

As argued above (example (11)), the first sentence is weakly rejected. However, ‘X or Y will be elected’ might well be true, and another speaker with different opinions on candidate Z’s chances might even agree with it. Thus, the coherent rejection of a sentence does not reduce to that sentence’s truth value.

<sup>12</sup> Recall that Restall uses different terminology. To him, acceptance and rejection are the *attitudes* corresponding to the *speech acts* of assertion and denial.



The differences between Restall's account and ours are more subtle. In his view, 'a statement is rejected [dissented from] if ... to accept [assent to] it would be a change of mind, and not merely a supplementation with new information' [Restall 2005: 196, fn. 6]. However, the speakers of (11) and (12) could come to assent to the rejected sentences simply as a result of getting *additional* information instead of *revising* any. So it seems that Restall would not include (11) or (12) as rejections.

Moreover, Restall has different goals than we do. He discusses logics whose formulae are not marked as asserted or rejected, whereas our aim is to explain how rejections can appear *as premisses and conclusions* in inferences. Restall discards this project because he finds it implausible to close assent under inference: never am I aware of *all* the consequences of what I assent to, so I cannot be said to have assented to them all. The move to commitment solves this: while I might not be aware of the consequences of my commitments, I can be held to them nonetheless. Once such a consequence is pointed out to me, I ought to accept it or admit to a mistake. In that sense, I am *implicitly* committed to the consequences of my commitments. In the next section, we will investigate what these consequences are.

### 3. Weak Rejectivist Logic

Our next task is to motivate a set of inference rules that constitute a logic of weak rejection and investigate its logical properties. We prove a classicality theorem, develop a sound and complete model theory and embed our logic into a modal logic of commitment.

#### 3.1 Inference Rules

We saw that the notion of public commitment is connected to that of dialogue coherence. We now use this fact as a guiding principle for finding inference rules. As we shall see, some rules of Rumfitt's [2000] bilateralist logic can be motivated on this basis. Other rules, however, will need to be weakened, since the Deduction principle turns out to be invalid for weak rejections. To partly make up for the loss in inferential strength, we add a new inference rule: the inference pattern in (5).

We use lowercase Latin letters to denote propositional atoms, uppercase Latin letters to denote propositional logic formulae, and lowercase Greek letters to denote force-marked formulae or absurdity. So,  $\varphi$  can be  $+A$ ,  $-A$  or  $\perp$ . The absurdity sign  $\perp$  is considered a punctuation mark and is therefore not force-marked [Tennant 1999; Rumfitt 2000]. We present our inference rules in a Gentzen-style natural deduction calculus. There are some sample derivations below.

**Preserving Commitment.** Our inference rules are about the necessary consequences of commitments with respect to coherence. That is, given that a speaker has undertaken certain commitments (displayed some attitudes), the rules tell us what *further* commitments that speaker is bound by on pain of incoherence. Since we linked commitment to coherence, this means that our inference rules *preserve commitment*. By definition, asserting and rejecting the same sentence is incoherent. Using  $\perp$  as a sign for

incoherence we can render this as follows.

$$\text{(Rejection)} \frac{+A \quad -A}{\perp}$$

Thus, a speaker's commitments constrain the space of coherent future speech acts. Conversely, we can draw conclusions about a speaker's commitments by observing which speech acts one can or cannot make coherently. Consider the following principle: if a speaker would be incoherent in asserting  $A$ , they are implicitly taken to have rejected  $A$ . This principle is a standard structural rule in bilateralist logics usually referred to as (half of) *Smilean reductio*.<sup>13</sup>

$$\text{(SR}_1\text{)} \frac{\begin{array}{c} [+A] \\ \vdots \\ \perp \end{array}}{-A}$$

Since asserting  $A$  would be incoherent, the speaker's extant commitments must be incompatible with them committing to  $A$ . This means that the speaker is already assumed to have displayed an attitude indicating dissent. That is, they have—explicitly or implicitly—rejected  $A$ . The dual principle also holds: if one *cannot later refrain* from committing to  $A$ , one is taken to *have already committed* to  $A$ .

$$\text{(SR}_2\text{)} \frac{\begin{array}{c} [-A] \\ \vdots \\ \perp \end{array}}{+A}$$

**Rejection and Negation.** Given our framework, a speaker can perform two speech acts involving  $A$ : they can assert  $A$  ( $+A$ , commit to  $A$ ) or weakly reject  $A$  ( $-A$ , refrain from committing to  $A$ ). Derivatively, we can put strong rejections as asserting  $\neg A$  ( $+\neg A$ , committing to  $\neg A$ ).<sup>14</sup>

We separate weak from strong rejections by observing that one might refrain from committing to  $A$  on grounds other than judging  $A$  false. A particular consequence of our view is that  $-A$  and  $-\neg A$  need not be incoherent, as  $A$  and  $\neg A$  might both be rejected for grounds other than falsity. However, we regard falsity as a way of being unassertible, so a strong rejection entails a weak one.

$$\text{(+}\neg\text{E.)} \frac{+\neg A}{-A}$$

<sup>13</sup> Being a structural rule, (SR<sub>1</sub>) is sometimes written as  $\Gamma, +A \vdash \perp \Rightarrow \Gamma \vdash -A$ .

<sup>14</sup> Dually, we might say that  $-\neg A$  is a *weak assertion*: it excludes a strong rejection in the same way that a weak rejection excludes a (strong) assertion. Weak assertions could be realized by, e.g., 'yes, I guess so' in (12). Our focus here is on rejection and how it compares to the embeddable operator 'not'. In ongoing work, we are investigating how such weak assertions compare to embeddable 'might'.

This rule has a dual ( $- \neg I.$ ), which we can now derive.  $+ \neg A$  entails  $-A$ , which is incoherent with  $+A$  by (Rejection). That is,  $+A, + \neg A \vdash \perp$ . Hence, by Smilean *reductio*,  $+A \vdash - \neg A$ .

$$(- \neg I.) \frac{+A}{- \neg A}$$

**Failure of Deduction.** A logic of weak rejection cannot validate the following principle of classical negation introduction:

$$\begin{array}{c} [+A] \\ \vdots \\ \perp \\ \text{(CNI)} \frac{\perp}{+ \neg A} \end{array}$$

Since  $-A, +A \vdash \perp$ , (CNI) would validate  $-A \vdash + \neg A$ . But then  $-$  would be a sign for strong rejection. Intuitively, it is clear why (CNI) must fail: inferring incoherence from an assertion only tells us that  $A$  is rejected, but  $+ \neg A$  tells us specifically that  $A$  is rejected *because* it is judged false. This also means that if the sign  $-$  denotes weak rejection, an application of ( $+ \neg E.$ ) incurs a strict loss of information. Dickie [2010] shows that (CNI) fails for strong rejections too: if  $-$  is a sign for strong rejection, then the Smilean *reductio* ( $SR_1$ ) is no longer valid, but (CNI) implies ( $SR_1$ ). A bilateralist appears unable to justify classical *reductio*. However, we will prove in the next subsection that our logic includes the following tautology.

$$\text{(CR)} \quad +((A \rightarrow (B \wedge \neg B)) \rightarrow \neg A)$$

This apparent tension is resolved by observing that asserting ‘ $A$  implies a contradiction’ again tells us *specifically* that  $A$  is rejected on those grounds. Hence, a proof of  $\perp$  from  $+A$  is a weaker statement than  $+(A \rightarrow (B \wedge \neg B))$  and thus (CNI) is a stronger principle than the above tautology (CR). More generally, this means that a logic of weak rejection must reject the Deduction principle: *asserting* that a sentence implies another is in general strictly more informative than a *derivation* of one from the other. Note that the argument does not rest on the *reductio* tautology (CR) being true: even if ‘ $A$  implies a falsity’ and ‘ $A$  is false’ are distinct grounds for rejecting  $A$ , either one of them is more specific information than just ‘ $A$  is unassertible’. Thus, rejecting the Deduction principle in a logic for weak rejection is not specific to our particular formalization.

We can, however, restrict attention to derivations with only asserted premisses. To this end, consider the following definition:

**Definition 3.1** (Subderivations). We write (a) for the assumption that there is a proof of  $\psi$  from  $\varphi$  and (b) for the assumption that there is a proof of  $\psi$  from  $\varphi$  where there are no undischarged rejected assumptions and all premisses used in the proof are of the form  $+A$ .

$$\begin{array}{cc} \varphi & \varphi \\ \text{(a)} \quad \vdots & \text{(b)} \quad +\vdots \\ \psi & \psi \end{array}$$

Note: (b) is still valid if there appears a formula of the form  $-B$  in the proof that is derived from asserted premisses.

This definition allows us to formulate the following weakened rules for the conditional:<sup>15</sup>

$$\begin{array}{c}
 [+A] \\
 +\vdots \\
 (+ \rightarrow I)^* \frac{+B}{+(A \rightarrow B)} \quad (+ \rightarrow E.) \frac{+(A \rightarrow B) \quad +A}{+B}
 \end{array}$$

We see no issue with  $(+ \rightarrow E.)$ , but  $(+ \rightarrow I.)$  requires some justification now. Recall that our inference rules are motivated in terms of commitment preservation. Now, announcing that one is refraining from committing is a commitment *about* another commitment: one commits to not committing.<sup>16</sup> From direct commitments to sentences, one cannot derive such a higher-order commitment, but only infer further commitments to sentences. Thus, the only way to derive a rejection from asserted premisses is by showing that there is a commitment to a *negated* sentence. So in the case of asserted premisses we are able to recover the information lost by applying  $(+ \neg E.)$  and are free to retain Deduction. This also licenses the following.

$$\begin{array}{c}
 +\vdots \\
 (+ \neg I.)^* \frac{-A}{+ \neg A}
 \end{array}$$

Thus, as long as the discourse starts exclusively from assertions, all rejected sentences are *strongly* rejected. But our logic makes room for rejections that are not assertions and are not reducible to assertions.

**Conjunction and Disjunction.** The following rules are valid for our account of weak rejection.

$$\begin{array}{c}
 (+ \wedge I.) \frac{+A \quad +B}{+(A \wedge B)} \quad (+ \wedge E._1) \frac{+(A \wedge B)}{+A} \quad (+ \wedge E._2) \frac{+(A \wedge B)}{+B} \\
 \\
 (+ \vee I._1) \frac{+A}{+(A \vee B)} \quad (+ \vee I._2) \frac{+B}{+(A \vee B)} \quad (+ \vee E.)^* \frac{+(A \vee B) \quad \varphi \quad \varphi}{\varphi}
 \end{array}$$

The rules on  $\wedge$  appear immediately reasonable. Regarding the  $\vee$ -introduction rules, one should take care not to be lead astray by pragmatics. Under some readings, the assertion

<sup>15</sup> Where we weaken one of Rumfitt's [2000] rules, we mark the rule with an asterisk.

<sup>16</sup> We formalize and further discuss this notion of refraining from committing in [subsection 3.4](#).

$A \vee B$  pragmatically entails that the speaker does not want to commit to either  $A$  or  $B$ , so moving from committing to  $A$  to committing to  $A \vee B$  to *not* committing to  $A$  might seem like a slip. We argue that the *derived* commitments are merely implicit in the dialogue and not themselves subject to pragmatics: a speaker who asserts  $A$  is *inferred* to commit to  $A \vee B$ , but never *said* ' $A$  or  $B$ '. Hence, implicatures should not be computed here. We restrict the minor premisses of  $(+\vee E.)^*$  because if  $A$  and  $B$  are rejected on grounds other than falsity, then one can reject both of them, but might still want to assent to their disjunction (see example (15)). Thus, it is not the case that  $-A, -B, +(A \vee B) \vdash \perp$ . The following derivation is admitted by unrestricted  $(+\vee E.)$  but not by  $(+\vee E.)^*$  because there are weak rejections in the minor premisses.

$$\frac{+(A \vee B) \quad \frac{-A \quad [+A]^1}{\perp} (\text{Rej.}) \quad \frac{-B \quad [+B]^1}{\perp} (\text{Rej.})}{\perp} (+\vee E.)^1$$

We can now derive the rules for rejected conjunctions and disjunctions.

$$\begin{array}{c} \begin{array}{ccc} (-\wedge I_1) \frac{-A}{-(A \wedge B)} & (+\wedge I_2) \frac{-B}{-(A \wedge B)} & (-\wedge E.) \frac{\begin{array}{c} [-A] \quad [-B] \\ \vdots \quad \vdots \\ \varphi \quad \varphi \end{array}}{\varphi} \end{array} \\ \\ \begin{array}{ccc} (-\vee E_1) \frac{-(A \vee B)}{-A} & (-\vee E_2) \frac{-(A \vee B)}{-B} & (-\vee I.)^* \frac{\begin{array}{c} +\dot{\quad} \quad +\dot{\quad} \\ -A \quad -B \end{array}}{-(A \vee B)} \end{array} \end{array}$$

The proofs are simple Smilean *reductios*. For instance,  $(-\vee E_1)$  can be derived as follows.

$$\frac{-\frac{(A \vee B) \quad \frac{[+A]^1}{+(A \vee B)} (+\vee I.)}{\perp} (\text{Rej.})}{\frac{\perp}{-A} (\text{SR}_1)^1}$$

**Smilean Inference.** We argued in the introduction that (5) is a valid inference pattern in its own right and presented valid natural language instantiations involving weak rejections. These examples in particular show that (5) is required to account for commitment preservation. Instead of reducing this pattern to rules on asserted premisses as Frege would want us to, we take Smiley at his word and regard it as a new mode of

inference altogether. Let us christen it *Smilean inference*.

$$\begin{array}{c}
 [+ \neg A] \\
 +: \\
 \text{(SI)} \frac{+ \neg B \quad -A}{-B}
 \end{array}$$

The subderivation states that in the restricted language of only asserted content, if one commits to  $A$  being false then one also commits to  $B$  being false. Smilean inference says that in that case it would be incoherent to refrain from committing oneself to  $A$  but not to  $B$ .

The addition of Smilean inference to the other rules is sufficient for our purposes. In particular, the pattern in (6),  $+(A \rightarrow B), -B \vdash -A$ , can be shown by Smilean *reductio*. Furthermore, we can derive the rules for rejected conditionals.

$$\begin{array}{c}
 +: \\
 (- \rightarrow \text{E.}_1) \frac{- (A \rightarrow B)}{-B} \quad (- \rightarrow \text{E.}_2)^* \frac{- (A \rightarrow B)}{+A} \quad (- \rightarrow \text{I.}) \frac{+A \quad -B}{- (A \rightarrow B)}
 \end{array}$$

**Definition 3.2.** We call the set of inference rules, except (CNI) and (CR), mentioned in this section so far *weak rejectivist logic*.

Smilean inference might appear *ad hoc* at first glance, but its merits can be appreciated by comparing it to the strong bilateralist inference rule  $(- \neg \text{E.})$ .

$$(- \neg \text{E.}) \frac{- \neg A}{+A}$$

Smilean inference can be understood as the appropriate weakening of  $(- \neg \text{E.})$  to weak rejections. That is, it is provably equivalent to the restriction of  $(- \neg \text{E.})$  to asserted premisses.

$$\begin{array}{c}
 +: \\
 (- \neg \text{E.})^* \frac{- \neg A}{+A}
 \end{array}$$

In this sense, Smilean inference completes the weak bilateralist picture by licensing the dual principle to  $(+ \neg \text{I.})^*$ . As we show below, this allows us to maintain the bilateralist defence of classical negation. However, while stipulating  $(- \neg \text{E.})^*$  might be seen as presupposing a classical semantics, Smilean inference is grounded solely in the linguistic data and justified in that it preserves commitment.

**Strong Bilateralism.** To conclude this section, we briefly discuss the bilateralist rules, as used by Rumfitt [2000], which are *not* valid for weak rejections.

$$\begin{array}{ccc}
 [+A] & & [+A] \quad [+B] \\
 \vdots & & \vdots \quad \vdots \\
 (+ \rightarrow \text{I.}) \frac{+B}{+(A \rightarrow B)} & (- \vee \text{I.}) \frac{-A \quad -B}{-(A \vee B)} & (+ \vee \text{E.}) \frac{+(A \vee B) \quad \varphi \quad \varphi}{\varphi} \\
 \\ 
 (+ \neg \text{I.}) \frac{-A}{+\neg A} & (- \neg \text{E.}) \frac{-\neg A}{+A} & (+ \rightarrow \text{E.}_2) \frac{-(A \rightarrow B)}{+A}
 \end{array}$$

We have argued above that the three rules in the first row are invalid for weak rejections. In particular,  $(- \vee \text{I.})$  is invalid for the same reasons as  $(+ \vee \text{E.})$ : the set  $-A, -B, +(A \vee B)$  is a coherent set of commitments, if, say, a speaker is in a state of epistemic uncertainty regarding the truth of  $p$  but knows that either  $p$  or  $\neg p$  must hold. The rules in the second row all express the hallmark of strong rejection: that falsity is the *only* ground for rejection.

### 3.2 Classicality

We now show that, in a defined sense, the logic of assertion is classical propositional logic. Smilean inference encodes a form of contraposition and this easily implies the classical axioms for negation.

**Lemma 3.3** (Contraposition).  $+(\neg A \rightarrow \neg B), +B \vdash +A$ .

**Proposition 3.4** (Classical Negation). The following can be derived:

- $\vdash +(A \rightarrow \neg \neg A)$ .
- $\vdash +(\neg \neg A \rightarrow A)$ .
- $\vdash +\neg(A \wedge \neg A)$ .
- $\vdash +(A \rightarrow (B \wedge \neg B)) \rightarrow \neg A$ .

In particular, then, negation behaves classically in weak rejectivist logic. We take this as a welcome result, if the goal of bilateralism is to justify classical patterns of inference. Furthermore, Proposition 3.4 immediately delivers:

**Theorem 3.5** (Classicality). Let  $\vdash^{\text{CPL}}$  be the derivability relation of classical propositional logic and  $\vdash$  be the derivability relation of weak rejectivist logic. It holds that  $A \vdash^{\text{CPL}} B$  iff  $+A \vdash +B$ .

The Classicality Theorem follows because the axioms for Classical Negation together with the introduction/elimination rules for asserted  $\vee, \wedge$  and  $\rightarrow$  give a standard axiomatization of propositional logic and, conversely, all weak rejectivist rules on asserted premisses are classically valid. Note that the Classicality Theorem means that we do not merely verify classical tautologies (as some non-classical logics also do, such as the Logic of Paradox [Priest 1979]), but in fact obtain the classical derivability relation.

This result might be surprising when considering Liar sentences. One might want to simultaneously refrain from committing to the Liar sentence, its negation and their

disjunction, but our logic says that it would be incoherent to do so, since  $+(A \vee \neg A)$  is a theorem. This theorem is derived using rules which have been motivated on the basis of the fact that they preserve commitment. Thus, if the semantic paradoxes require a revision of classical logic, this will be because they call into question the claim that the rules involved in the derivation of the Classicality Theorem preserve commitment, and discussion should centre around these rules.<sup>17</sup> Note, however, that there are treatments of the semantic paradoxes on which both the Liar sentence and its negation should be (weakly) rejected, while their disjunction should be asserted (see, for instance, McGee [1990]). Thus, the semantic paradoxes are not immediate counterexamples to the logic of rejection developed here.

### 3.3 Semantics for Weak Rejections

We now give a semantics for which weak rejectivist logic is sound and complete. In strong bilateralist logics, a classical valuation is *correct* for  $+A$  if  $A$  holds, and for  $-A$  if  $A$  fails. Now, an  $\omega$ -pointed model is a sequence of  $\omega$ -many classical valuations. It is correct for  $+A$  if  $A$  holds at every point, and for  $-A$  if  $A$  fails at some point. This formalizes the intuition that there are many ways for a formula to be unassertible, and any of them is grounds for rejecting it.

**Definition 3.6.** An  $\omega$ -pointed model is a mapping  $V$  from  $\omega$  to models of propositional logic.

- For any  $x \in \omega$ ,  $V \models_x A$  iff  $V(x) \models A$  (as a model of propositional logic).
- Never  $V \models \perp$ .
- $V \models +A$  iff  $\forall x \in \omega: V \models_x A$ .
- $V \models -A$  iff  $\exists x \in \omega: \text{not } V \models_x A$ .

**Theorem 3.7** (Soundness). Weak rejectivist logic is sound on  $\omega$ -pointed models.

*Proof sketch.* This is a standard induction on the length of derivations. We present the inductive step for a rule involving restricted antecedents,  $(+\neg I)^*$ :

Assume  $\Gamma \vdash^D +\neg A$  by an application of  $(+\neg I)^*$ . That is,  $\Gamma \vdash^{D'} -A$  where  $D'$  uses only asserted premisses from  $\Gamma$ . Let  $\Gamma'$  be the asserted formulae in  $\Gamma$ . Then  $\Gamma' \vdash^{D'} -A$ . Assume that not  $\Gamma \models +\neg A$ . Then there is a model  $V$  of  $\Gamma'$  and a point  $y \in \omega$  such that  $V \models_y A$ . Construct an  $\omega$ -pointed model  $V'$  where every point is  $y$ : for any  $x$  and atom  $p$ ,  $V' \models_x p$  iff  $V \models_y p$ . Since  $\Gamma'$  contains only asserted formulae,  $V' \models \Gamma'$ . Also, since  $V \models_y A$ ,  $V' \models +A$ . But by induction,  $V' \models -A$ . Contradiction.

To prove completeness, we derive the following lemma from the Classicality Theorem.

**Lemma 3.8.** Let  $\Gamma$  be a consistent set of only asserted formulae. Assume  $\Gamma \cup \{-A\}$  is consistent. Then the set  $\{-A\} \cup \{B : +B \in \Gamma\}$  is satisfiable in classical propositional logic.

<sup>17</sup> There is work to be done in comparing weak rejectivist logic with the paraconsistent logics of Field [2008]. In them too, the Deduction principle fails for a class of defective premisses, but remains valid on a classical fragment of non-defective premisses. Field also distinguishes conditionals from derivations as we did in our discussion of (CNI) vs (CR). However, Field has it that for a class of defective  $A$ s,  $A \vee \neg A$  must fail. This is not the case for weak rejectivist logic.



**Theorem 3.9** (Completeness).  $\Gamma \models \varphi$  implies  $\Gamma \vdash \varphi$ .

*Proof sketch.* Given a set of formulae consistent in weak rejectivist logic, we can construct a model by letting every rejected formula fail at a single point.

### 3.4 Rejectivist Logic as Modal Logic

We now show that KD45 modal logic is also a semantics for which weak rejectivist logic is sound and complete. That is, given a suitable translation, the semantic consequence relation  $\models^{\text{KD45}}$  coincides with the derivability relation  $\vdash$  of weak rejectivist logic. This further vindicates from a formal point of view our informal discussion, which justified inference rules in terms of commitment preservation.

**Modal Translation.** In a modal logic of public commitment (where  $\Box\varphi$  stands for a commitment to  $\varphi$ ) we can phrase ‘committing to  $A$ ’ as  $\Box A$  and publicly refraining from committing to  $A$  as  $\Box \neg \Box A$  (read: a commitment to a non-commitment). Note that refraining from committing is not  $\neg \Box A$ , which says that a speaker is *uncommitted* to  $A$ . This is certainly true of many sentences, but rejecting a sentence has *public* consequences. Translating a weak rejection of  $A$  to  $\Box \neg \Box A$  makes this explicit.

**Definition 3.10.** Let  $\varphi$  be a formula of weak rejectivist logic. The *modalization* of  $\varphi$  is defined as:

- $\varphi^m = \perp$ , if  $\varphi = \perp$ ;
- $\varphi^m = \Box A$ , if  $\varphi = +A$ ;
- $\varphi^m = \Box \neg \Box A$ , if  $\varphi = -A$ .

Let *modalized weak rejectivist logic* be the inference rules of weak rejectivist logic where premisses and conclusions have been modalized.

Asher and Lascarides [2008] model public commitment as a K45 modality. We show below that weak rejectivist logic is sound and complete for KD45 modal logic. Asher and Lascarides do not include (D) to account for the fact that, sometimes, natural language speakers can have incoherent commitments. Although this is true as a matter of fact, the implicit commitments of incoherent speakers cannot be computed. Thus, we require the inclusion of (D). For reference, these are the axioms of KD45.

$$(K) \Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi) \quad (D) \Box\varphi \rightarrow \neg \Box \neg \varphi$$

$$(4) \Box\varphi \rightarrow \Box\Box\varphi \quad (5) \neg \Box\varphi \rightarrow \Box \neg \Box\varphi$$

Asher and Lascarides justify the K45 axioms as follows. Axiom (K) commits a speaker to the consequences of their commitments. Axioms (4) and (5) ensure that commitments respect coherence: without (4), a speaker could undertake a commitment,  $\Box A$ , but also commit to not having undertaken that commitment,  $\Box \neg \Box A$ —but this would be incoherent.<sup>18</sup> Axiom (5) works in the same way, but prevents speakers from denying

<sup>18</sup> Asher and Lascarides would here infer  $\Box \perp$ , i.e., that the speaker simply is incoherent. Our inclusion of (D) makes  $\Box \neg \Box A$  and  $\Box\Box A$  mutually exclusive.

not having made a certain commitment. That is, if one did not undertake a commitment, they cannot claim to have done otherwise.<sup>19</sup>

It might appear counterintuitive that (5) allows one to infer from a *non*-commitment that one has *refrained* from committing. However, this is because modal logic is *static* whereas dialogue is *dynamic*. That is, we compute the implicit commitments of a speaker at a fixed point in time. At a fixed point, one is either committed to a sentence or not—and, as explained above, committed to that state. But if the dialogue moves on, commitments change. In the dialogue *dynamics*, the difference between non-commitment and refraining from committing becomes important.

This can be appreciated by considering the Smilean *reductios* and (Rejection), which are about how commitments can be coherently updated. That is, they tell us how one's current state constrains the coherence of future speech acts. They modalize as follows.

$$(SR_1) (\Box A \rightarrow \perp) \rightarrow \Box \neg \Box A \quad (SR_2) (\Box \neg \Box A \rightarrow \perp) \rightarrow \Box A$$

$$(Rej.) (\Box A \wedge \Box \neg \Box A) \rightarrow \perp$$

It can be shown that these three principles axiomatize the class of KD45 frames, but would not do so if we were to map  $\neg A$  to  $\neg \Box A$ . Thus, the difference between non-commitment and refraining from committing matters dynamically.

**Soundness and Completeness.** We now show that modalized weak rejectivist logic is sound and complete for KD45. Definition 3.11 is the modalized pendant to asserted premisses, and the following technical lemma represents the only challenge in the soundness proof.

**Definition 3.11.** Call a modal formula  $\varphi$  *necessitated* if it is of the form  $\varphi = \Box A$  for a propositional formula  $A$ .

**Lemma 3.12.** Let  $\Gamma$  be a set of necessitated modal formulae. Let  $A$  and  $B$  be propositional formulae. If  $\Gamma, \Box A \models^{KD45} \Box B$  then  $\Gamma \models^{KD45} \Box(A \rightarrow B)$ .

*Proof sketch.* By contraposition. If there is a model of  $\Gamma$  with a witness for  $\neg \Box(A \rightarrow B)$ , one can construct a model of  $\Gamma, \Box A$  and  $\neg \Box B$  by duplicating the witness as in the proof of Theorem 3.7.

**Theorem 3.13** (Soundness). If  $\Gamma$  is a set of formulae of weak rejectivist logic such that  $\Gamma \vdash \varphi$ , then  $\{\psi^m : \psi \in \Gamma\} \models^{KD45} \varphi^m$ .

Modal operators can embed, while the force-markers  $+$  and  $-$  cannot, so the language of modal logic is more expressive. Thus, one might worry that by modalizing weak rejectivist logic we gain theorems that we did not have before. To show that this is not the case, we first define a back-translation from modal logic to weak rejectivist logic. Since  $+$  and  $-$  do not embed, this mapping is only partial. We then establish a completeness result.

**Definition 3.14.** Call a formula  $\varphi$  of modal logic *rejectivist* if there is a  $\psi$  in weak rejectivist logic with  $\psi^m = \varphi$ . For the inverse mapping write  $\varphi^r = \psi$ .

<sup>19</sup> Parts of this paragraph are based on conversations with Alex Lascarides on the topic.

**Theorem 3.15** (Completeness). Let  $\Gamma$  be a set of rejectivist modal formulae. If  $\Gamma \models^{\text{KD45}} \varphi$  and  $\varphi$  is rejectivist, then there is a proof of  $\varphi^r$  in weak rejectivist logic from the premisses  $\{\psi^r : \psi \in \Gamma\}$ .

*Proof sketch.* One can use the completeness result on  $\omega$ -pointed models and transform an  $\omega$ -pointed model into a KD45-model by letting the accessibility relation be  $\omega \times \omega$ . The translation between the two logics works as expected.

We have established that weak rejectivist logic is sound and complete for KD45 when the force-markers are translated into the modal operators  $\Box$  and  $\Box \neg \Box$ .<sup>20</sup> Since there are independent arguments justifying KD45 as the logic of coherent commitment, we take this to confirm that the inference rules of weak rejectivist logic are commitment preserving.

#### 4. Recovering Strong Rejection

In the modal characterization of weak rejectivist logic, an assertion of  $A$  is  $\Box A$ , a weak rejection is  $\Box \neg \Box A$ , and a strong rejection can be put as  $\Box \neg A$ . Note that under KD45,  $\Box A$ ,  $\Box \neg A$  are *contrary* whereas  $\Box A$ ,  $\Box \neg \Box A$  are *contradictory* in the classical square of opposition. This aligns in terminology with the distinction between internal (narrow-scoped) and external (wide-scoped) rejection discussed by Rumfitt [1997].

Linguistic evidence supports the claim that natural language has a general preference for contrary negation over contradictory negation [Horn 1989: chs 4–5]. The most familiar case is the *neg-raising* phenomenon: when a speaker is stating certain attitudes (such as belief) towards a proposition, a syntactically wide-scoped negation can be *raised* to take narrow scope. For example, in ‘I don’t believe  $A$ ’, a literal interpretation could be  $\neg \mathbf{B}A$  (with a belief modality  $\mathbf{B}$ ): ‘the speaker lacks the belief that  $A$ ’. However, the typical reading of such an utterance is ‘the speaker believes that not- $A$ ’, which is  $\mathbf{B} \neg A$ .

We contend that this preference also applies to strong vs weak rejections. Consider the following variants:

- (16) Did Homer write the *Iliad*? No, he did not.
- (17) Did Homer write the *Iliad*? No, Homer did not exist.
- (18) Did Homer write the *Iliad*? No.

Example (16) is a strong rejection, committing the speaker to *Homer did not write the Iliad*, whereas (17) is a weak rejection that does not entail any such commitment. The bare ‘no’ in (18) might look ambiguous between the two, but it seems to us that it has the conversational effect of (16), and that the reading in (17) arises because of the added sentence ‘Homer did not exist’. In that sense, strong rejections are the preferred readings of bare ‘no’. ‘Preferred’ means that they are read as strong rejections by default, but this reading is usually cancellable: (17) is a continuation of (18) that cancels the preferred reading.

Our proposal now enables us to give a straightforward account of rejection in natural language: all rejections are weak rejections, but there is a preference to read

<sup>20</sup> The proof of Theorem 3.15 suggests that weak rejectivist logic is sound and complete for S5 modal logic. However, one cannot phrase the reflexivity axiom  $\Box A \rightarrow A$  in the bilateralist language.

(contradictory) dissent as (contrary) assent to a negative.<sup>21</sup> Equivalently, the default reading of unassertibility is falsity. If we encode this preference as a rule of inference, we derive strong bilateralist logic.

**Theorem 4.1.** Weak rejectivist logic plus the following inference rule is equivalent to bilateralist logic.

$$(+ \neg I.) \frac{-A}{+ \neg A} \quad \text{respectively,} \quad \frac{\Box \neg \Box A}{\Box \neg A}$$

Strictly speaking, this move is unsound:  $(+ \neg I.)$  encodes the preferred reading as non-cancellable. Nonetheless, one can now think of strong bilateralism as being about the preferred interpretation of bare ‘no’ and the default interpretation of unassertibility. As such, it can reclaim linguistic plausibility.

## 5. Conclusion

We have addressed a worry from the prior literature on rejection: that rejections should not or cannot be premisses in inferences. We have shown that weak rejections are suitable premisses by presenting appropriate examples and developing a corresponding inferential system. This system comes with its own model theory in the form of  $\omega$ -pointed models, but is also complete for the modal logic KD45.

On our account, weak rejection is a primarily dialogical phenomenon, characterized in terms of public commitment. This approach is inspired by the formal semantics literature on agreement and disagreement in dialogue. The normative principle justifying our inference rules is the maxim to *maintain coherence* in a dialogue. Hence, we do not need to appeal to the truth or falsity of the asserted or rejected sentences. This allows us to account for cases in which a sentence is coherently rejected even if true and not unassertible in principle. On the basis of considerations concerning the preservation of commitment with respect to coherence, we are also able to motivate the Smilean *reductio* rules, which had to be stipulated in previous literature.

The inferential system for weak rejection we propose generalizes on both the Fregean view and the Smilean bilateralist view. The system reduces to Smiley’s bilateralist logic if weak rejections are not present in the dialogue, but accounts for their occurrence by only including the bilateralist rules from the prior literature that are still valid for weak rejections. The singular addition we make to these rules is the *Smilean inference*: an inference pattern endorsed by Smiley and entertained by Frege; the latter at least insofar as he recognizes some natural language instances for the purposes of his argument. We have established that the logic is classical on its asserted fragment, despite the fact that, as we observed, a logic of weak rejection cannot verify the full Deduction principle.

Finally, by considering a general linguistic preference for contrariness, we can give a full account of the rejection phenomenon, both strong and weak, in dialogue. We conclude that strong bilateralist logics can be understood as being about the linguistically

<sup>21</sup> There are competing proposals for the *mechanism* that realizes this preference [Horn 1989: ch. 5]. We need not take a stand, since any mechanism accounting for the linguistic data would serve our purposes.

preferred interpretations of rejections. Hence, we consider them vindicated from earlier attacks on their linguistic plausibility.<sup>22</sup>

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